A brief history of polynomial identities up to the generalized case

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Let G be a finite group, A and W be associative G-graded algebras over a field F of characteristic zero. We say that A is a G-graded W-algebra, if A is a W-bimodule such that

$$w(a_1a_2) = (wa_1)a_2, (a_1a_2)w = a_1(a_2w), (a_1w)a_2 = a_1(wa_2)$$

for any $w \in W, a_1, a_2 \in A$, and

$$av \in A^{hg}, \quad va \in A^{gh}$$

for any $v \in W^g,\ a \in A^h,\ h, g \in G$.

Let $W\langle X\rangle^G$ the free G-graded W-algebra. A G-graded generalized polynomial W-identity, or symply graded generalized identity, of A is an element $f = f(x_1^{g_1}, \dots, x_{t_1}^{g_1}, \dots, x_1^{g_m}, \dots, x_{t_k}^{g_m})$ of $W\langle X\rangle^G$ that vanishes under all evaluations $x_i^{g_j} \to a_{g_j,i} \in A^{g_j}$. The graded generalized identities are a natural generalization of polynomial identities, arising when $W = F$ and the grading is trivial.

The purpose of this talk is to give an overview on the graded generalized identities and their growth, starting by reviewing the basic results of the theory of polynomial identities.